## EXCAMPLE FORMAT for Part II Physics in 2023

EXAMPLE 9.00 to 11.00

PHYSICS (7) PHYSICAL SCIENCES: HALF SUBJECT PHYSICS (7)

Quantum Condensed Matter Physics - EXAMPLE

### Candidates offering this paper should attempt all questions.

The approximate number of marks allocated to each question or part of a question is indicated in the right margin. This paper contains six sides, including this coversheet, and is accompanied by a handbook giving values of constants and containing mathematical formulae which you may quote without proof.

STATIONERY REQUIREMENTS  $2 \times 20$  Page Answer Book

Rough workpad Yellow master coversheet

# SPECIAL REQUIREMENTS Mathematical Formulae handbook Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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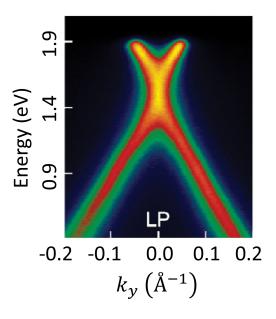
SECTION A Answers should be concise and relevant formulae may be assumed without proof.

1 State the form of the wavefunction for an electron in a crystal. What is meant by the crystal momentum of the electron and how does it differ from the true momentum of the electron? [4]

2 Give a brief explanation of the concepts of drift velocity, electron mobility, and effective mass, as used in solid state physics.

Consider the Stoner instability of free conduction electrons of total density  $n = n_{\uparrow} + n_{\downarrow}$  in a metal with Fermi energy  $E_F$  and density of states  $g(\epsilon)$ . Electrons of opposite spin interact via a short-range repulsive interaction U, giving rise to a total interaction energy of  $E_{IA} = Un_{\uparrow}n_{\downarrow}$ . Derive an expression for the critical value  $U_c$  of the interaction, above which a ferromagnetic state has a lower energy compared to an unpolarised state with  $n_{\uparrow} = n_{\downarrow}$ . [4]

4 The picture below shows an ARPES measurement of a doped graphene sample. Describe how one can extract the Fermi energy and Fermi velocity from this measurement and obtain approximate values.



[4]

[4]

#### SECTION B

- 5 This question is about the optical response of insulators and simple metals. We will start with *insulating* materials, where bound electrons respond to an optical light field  $E = E_0 e^{-i\omega t}$  according to the Lorentz model.
  - (a) State the relevant differential equation for the oscillating displacement  $u(t) = u_0 e^{-i\omega t}$  considering a single atomic transition of frequency  $\omega_0$  and explain the physical origin of all the terms.

[3]

(b) Show that this differential equation gives rise to the following frequency-dependent permittivity,

$$\epsilon_{\omega} = 1 + n \frac{q^2}{m\epsilon_0(\omega_0^2 - \omega^2 - i\omega\gamma)},$$

where *m* and *q* denote the mass and charge of the electron,  $\epsilon_0$  is the vacuum permittivity, *n* the density of electrons, and  $\gamma$  is a phenomenological damping rate. [5]

(c) Sketch the real and imaginary parts of the permittivity and discuss their physical significance in the case of a weakly absorbing medium. [3]

(d) Repeat the above sketch for the case of several atomic transitions and discuss the connection to normal and anomalous dispersion in optics. [3]

From here on, consider the situation of a simple metal where the conduction electrons can be seen as free electrons.

(e) Discuss how the model above needs to be modified for simple metals and sketch the resulting permittivity. [3]

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6 Start by considering a homogeneous bulk semiconductor in three dimensions. Assume that the dispersion relations for electrons in the conduction band and holes in the valence band are given by

$$\epsilon_c(k) = \epsilon_c + \frac{\hbar^2 k^2}{2m_e^*}$$
 and  $\epsilon_v(k) = \epsilon_v - \frac{\hbar^2 k^2}{2m_h^*}$ .

(a) Show that the densities of states for those bands per unit volume are given by [4]

$$g_e(\epsilon) = \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} (\epsilon - \epsilon_c)^{1/2} \quad \text{and} \quad g_h(\epsilon) = \frac{1}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2}\right)^{3/2} (\epsilon_v - \epsilon)^{1/2}.$$

(b) Use the above results to show the law of mass action, i.e., show that the product of intrinsic carrier concentrations  $n_i^2 = n p$  is given by

$$n_i^2 = \frac{1}{2} (m_e^* m_h^*)^{3/2} \left(\frac{k_B T}{\pi \hbar^2}\right)^3 \exp\left(-\frac{\epsilon_c - \epsilon_v}{k_B T}\right),$$

where *n*(*p*) denotes the density of electrons (holes). You can assume the semiconductor to be non-degenerate, i.e., the Fermi distribution can be approximated to  $f(\epsilon) \approx \exp\left(-\frac{\epsilon-\mu}{k_BT}\right)$ .

(c) Discuss why semiconductor devices can operate only in a limited temperature regime.

(d) Sketch the current through a pn junction as a function of applied voltage and discuss how the temperature dependence of  $n_i^2$  calculated above can be extracted from such measurements.

(e) Discuss why the Hall coefficient can change sign as a function of temperature in doped semiconductors.

You might find the following integrals useful:  $\int_0^\infty \sqrt{x}e^{-x}dx = \sqrt{\pi}/2$ ,  $\int_0^\infty x^2 e^{-x^2}dx = \sqrt{\pi}/4$ .

## END OF PAPER

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[5]

[3]

[3]

[2]